

## COMP523 Tutorial 3

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### Problem 1

Show that if a graph  $G$  has no cycles of odd length, then it is bipartite.

### Problem 2

Prove that a connected graph with  $n$  nodes has at least  $n - 1$  edges.

### Problem 3

Prove the following property for the layers produced by BFS: For any edge  $(u, v)$ , either  $u$  and  $v$  are in the same layer, or  $|L(u) - L(v)| = 1$ , where  $L(x)$  is the layer of node  $x$ .

### Problem 4

Let  $G = (V, E)$  be a connected graph and let  $s \in V$  be a node of  $G$ . Suppose that we run  $\text{DFS}(G, s)$  and obtain a DFS spanning tree  $T$  and that we also run  $\text{BFS}(G, s)$  and obtain the same BFS spanning tree  $T$ . Prove that  $G = T$ .

### Problem 5

A *Hamiltonian path* in a DAG  $G$ , is a path that visit all the nodes of the graph *exactly once*. Prove that a Hamiltonian path in a DAG  $G$  exists if and only if  $G$  has a *unique* topological order.