COMP523 Tutorial 9 - Solutions

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Problem 1

Kleinberg and Tardos, Algorithm Design, Chapter 13, Exercise 1.

Solution

Assume that we have the three colours {red, white, blue}. Consider the following simple algorithm: For each vertex, colour it red with probability 1/3, white with probability 1/3 and blue with probability 1/3. We will prove that the approximation ratio of this algorithm is 3/2.

For an edge e, let Y_e be a random variable which is 1 if the edge is satisfied and 0 otherwise. Let X be random variable, denoting the number of satisfied edges; by definition, it holds that

$$X = \sum_{i=1}^{|E|} Y_e.$$

We are interested in the expected value of X. We have:

$$\mathbb{E}[X] = E\left[\sum_{i=1}^{|E|} Y_e\right] = \sum_{i=1}^{|E|} E[Y_e] = \sum_{i=1}^{|E|} \Pr[e \text{ is satisfied,}]$$

where the second equation follows from the linearity of expectation. The probability that an edge e is satisfied is equal to the probability that its two endpoints receive different colours, which can be easily seen to be 2/3. Therefore, we have that

$$\mathbb{E}[X] = \frac{2|E|}{3}.$$

By using a trivial bound of $|E|$ on the number of satisfied edges in the optimal solution, we obtain the desired 3/2 approximation.

Problem 2

Kleinberg and Tardos, Algorithm Design, Chapter 13, Exercise 7.

Solution

The solution can be found at Williamson and Shmoys, The Design of Approximation Algorithms, Chapter 5.3.

Link: `https://www-cambridge-org.liverpool.idm.oclc.org/core/services/aop-cambridge-core/content/view/337E633A6859743A53EE05AC8DD1162D/9780511921735c5_p99-136_CBO.pdf/random_sampling_and_randomized_rounding_of_linear_programs.pdf`